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Abstract
This paper shows that the post-Walras general equilibrium theory is irrelevant to real contemporary economic life.

The main achievement of modern General Equilibrium Theory is the proof of equilibrium’s existence. It might be that the proof of the equilibrium existence is a mathematical achievement, but the question is whether these proofs are harmonious with the economic situation in reality.

This paper traces concisely how Walras’s theory has been causing economic science to deviate in an erroneous direction and reaching a deep crisis; because post-Walras’s economists, since Pareto, have misunderstood and misinterpreted Walras’s economic theory. This group of Post-Walras authors (Pareto, Cassel, Schlesinger, Wald, and von-Neumann, Hicks, Keynes, Lange, and Patinkin) then recast Walras’s theory into incorrect and wrong form; their error further compounded when a later group of economist-mathematicians (Arrow, Debreu, Friedman, Samuelson, Solow and others) accepted their interpretation without reservation.

Post-Walras’s economists ignore Walras’s less known assumptions and blame him for disregarding the problem of equilibrium existence, uniqueness and stability and comparative-static. Therefore, their main objective since the beginning of the 20th century was the rigorous proof of equilibrium existence. However, this proof was based on unrealistic assumptions and along the road the goal of economics was lost.

The nine crucial, unrealistic assumptions will be considered and will illustrate that modern general equilibrium theory is irrelevant to real economics and is also far removed from Walras’s general equilibrium theory.
1. Introduction

One of the main causes of the current financial-economic crisis is a crisis of economic science, which has been occurring from the beginning of the last century, namely, after the Neoclassical Economic School (Walras, Marshall). Despite the fact that the real economic life over the past century has greatly improved, state of economic science is deteriorated. Almost all of the topics of economic theory or degraded, or are at the beginning of the last century. Nevertheless, it is very difficult to call a new paradigm-rich modern economic theory to replace it classical or neoclassical paradigm. This statement may seem very strange, considering the "revolution in economics" (Keynesian, monetarist and rational expectations) and the fact that every year there is new Nobel Prize in Economic Sciences. This statement seems even more strange in light of the emergence of useful tools such as mathematical (linear) programming and other areas of operations research (especially game theory), econometrics, and, of course, powerful and "smart" computers.

The debate on the methodology of economics between theorists, economists, philosophers and historians of economics economic thought suggests that there is still no consensus on the methodology of economics. One group of authors argues that the theory should be close to the economic reality as possible. At the same time, another group of authors suggests that unrealistic assumptions character determines the measure of the significance of the theory. We support the first approach.

There are the following three statements: first, to be an inverse relationship between theory and reality; Second, the abstract theory means simplification of the real economy by assumptions; and third, the main objective of such a theory should be its practical implementation. The last statement is closely related to the previous as if the abstract theory distorts reality its use is not possible. In other words, there must be an interconnection between abstract theory and reality. Moreover, the relationship between theory and reality should serve as a criterion whether there is progress (development) or regression (degradation) in economic science. Classics (Smith, Ricardo, Marx) and Walras, Marshall suggested that the theory should be as close as possible to the economic reality.

The "abstract method" means that the theory reproduces the real economic life only in a simplified form, so that the theory should not contradict reality, and directed it. It is clear that the abstract theory can never reproduce reality exactly, but should be as close as possible to it (Marshall, p.1).

For example, Walras’s theory is characterized by free competition, uniform prices, but does not address the public sector (taxation) and international trade. At the same time, in Walras’s approach in equilibrium voluntary unemployment may exist with a positive price. In the post-Walras approach (including the present), this price is zero, which contradicts the reality (Arrow and Hahn; Debreu, 1959; Mas-Colell and et. Al). As a result, we are not "abstract method" and absurd situation.

The scientific method is also characterized by an evolutionary approach, which means compatibility between the progress of human society and economic theory; therefore, the paradigm should be changed in accordance with the development of the real economy in such a way as to ensure compatibility between them.

Thus, in order to establish whether there is either degradation or progress of economic science for a specific period of time, it is necessary to compare the mathematical model and assumptions of the beginning and end of the period that allows you to see whether there is a new paradigm.

Those who argue that there is a significant improvement in the general equilibrium theory (Negishi, 1989; Weintraub, 1985 and 2002) and those who are skeptical about the theory of general equilibrium (Rosenberg, 1983) are wrong. Namely, they have misinterpreted and misunderstood Walrasian general equilibrium theory (Davar, 1994, 2012 and 2014b). Recent
publications in the economic literature regarding the Walrasian general equilibrium theory (Bridel, 1997; Van Daal and Jolink, 1993; Walker, 1996 and 2006) do not shed light on the true interpretation of the Walrasian approach. As a result, all statements about the progress or regression of economic theory are questionable.

The paper consists of this introduction, three sections and a conclusion. The second section, Walras’s method of general equilibrium theory is briefly discussed. The third section, concise story how Walras’s theory became irrelevant to reality is considered. The fourth section deals with unrealistic assumptions of modern general equilibrium theory. Finally, conclusion is presented.

2. Walras’s Method of General Equilibrium Theory

Let's discuss very briefly Walras’s method of establishment and re-establishment of equilibrium. Walras used the common method of equilibrium establishment and re-establishment (variation of prices) in the four types of economies (Davar, 1994 and 2014b).

The first economy, the Exchange Economy, deals with the problems of the exchange of consumptions’ goods, namely the problems encountered when determining the equilibrium prices of consumable goods. Walras formulated a law to establish equilibrium prices for the exchange economy.

The second, Production Economy, is an enlarged version of the exchange economy. Namely, productive services (labor, fixed capital and land) are combined to produce consumers’ goods. Therefore the problem of establishing the equilibrium price of services is discussed together with the price of goods for consumption. Furthermore the conditions of the law of equilibrium for the production economy, combines the law of the exchange economy (equilibrium prices of consumption’s goods) with the law whereby the services’ equilibrium prices are established.

Walras formulated the third type of economy, Capital Formation and Credit, by enlarging the production economy by adding the production of new capital goods (investment) and saving. This means that at this step in his theory the problems of new capital goods production and their subsequent demand in parallel with the problems of creating saving are additional issues of capital formation. The general law of equilibrium of capital formation formulated by the law of capital formation and combined with the law of production.

The fourth type of economy, Circulation and Money, is enlarged by the problems of Capital formation and credit. Here Walras added the problems of the determination of prices and quantities for raw materials, circulation capital goods and money for circulation. For this purpose Walras considered problems of determination of the demand for money for circulation for both consumption and production, and of supply of money for the production. So, the law of circulation and money is obtained by combining the law of capital formation and credit with the law of circulation and money. Therefore, this law includes the laws of all types of economies.

It could be said that the term “money” makes an illusion that in Walras’s previous economies money is absent. However this is incorrect, because a numéraire fulfills all the three main functions of money (standard of measure, tool of exchange, and store of value), which might be expressed by means of any good (from goods for consumption), and is attributed to all the various types of economies of Walras (Walras, 1954, p.189). Therefore, in the fourth type of economy (circulation and money) together with the numéraire Walras used fiat money, as a specific character of the last economy and employing it as money for circulation. So, in the last type of economy Walras considered two kind of money: money commodity (numéraire) and fiat money.

Walras first considered the problem of establishing equilibrium for given basic data for the economy of individual (utility functions for each commodity and services separately, and available quantities of goods and services). Determination of the supply and demand for goods
and services for each individual economy is the first step for the random price system. Yet, either 
the offer of or demand for a commodity used as the *numéraire* depends on the balance between 
the total value of demand (expenditure) and the total value of offer (income) of the commodities 
not used as the *numéraire*. The total supply and demand of goods and services may be calculated 
from the results of models of individuals’ economies.

At this stage, for the entire economy, Walras formulated two models (equation system) 
for the equilibrium state and the disequilibrium state, and described the process of establishment 
of equilibrium by means of the *tâtonnement* algorithm (Davar, 1994, 2002, 2005, 2014b; Negishi, 
1985, pp.170-3; Van Daal and Jolink). Namely, Walras shows how this iterative process 
transforms any initial disequilibrium situation to the equilibrium situation if it is possible, and by 
this, guarantee its solvability. The each isolated iteration of *tâtonnement* is divided into two 
stages: firstly, equilibrium establishment for a certain good (or service) – *partial equilibrium*; and 
secondly, general equilibrium establishment for all categories simultaneously – *general equilibrium*. Walras asserted that the partial equilibrium of a certain category would be exist if the 
essential assumptions plus the additional requirement, that is, the total (aggregate) demand curve 
and the total offer curve have *at least one intersections point* (Walras, 1954, pp.108 and 171). It 
must be stressed that there are economists who has been claiming that Walras’s *tâtonnement* is 
the process of adjustment only of prices without of quantities (Leijonhufvud, p. 99). Yet, in 
Walras’s approach, *there might be voluntary unemployment; moreover, according to Walras’s approach also might be considered “forced unemployment”*. 

Walras concentrated throughout on the *Law of Equilibrium State*, which is different from 
the well-known “Walras’ law” formulated by his followers (Morishima; Davar, 1994 and 2012). 
While the Law for more advanced economies only applies to new markets entering the system, it 
automatically includes the law relating to earlier types of the economy. For example, the Law of 
Capital Formation and Credit only relates to new capital goods, saving, investment and rate of 
income. Thus the equilibrium law for consumer goods and services for earlier types of economy 
(exchange and production economies) is integrated into the law for the economy in question 
(capital formation and credit).

Moreover, Walras discussed the variation of prices, or re-establishing the equilibrium 
following changes in the given basic data for an individual or group. This means that if any 
individual discovers that in the equilibrium state his services (or goods) are not traded, he might 
changes his basic data according to the results of obtained equilibrium state (Ingrao and Israel). 
Then, the new process of equilibrium establishment is required.

When Walras’s approach is discussed in the post-Walrasian literature, unfortunately, the 
both models of individual’s (micro) and entire (macro) economy is presented either through the 
exchange economy (in general), or through the production economy. Moreover, this 
representation of Walras's model differs from his original model and is incomplete.

Finally, Walras, as well as Marx, stated repeatedly that in reality equilibrium never 
achieved. At the same time, Walras asserted that study of equilibrium achievement it is necessary 
to managing real economics, by revealing the nature of distortion of equilibrium state and 
treatment them. Walras stated that:

Such is continues market, which is perpetually tending towards equilibrium without ever 
actually attaining it, because the market has no other way of approaching equilibrium 
except by groping , and before the goal is reached, it has to renew its efforts and start 
over again , all the basic data of the problem, e.g. the initial quantities possessed, the 
utilities of goods and services, the technical coefficients, the excess of income over 
consumption, the working capital requirements, etc., having changed in the meantime. 
Viewed in this way, the market is like agitated by the wind, where the water is 
incessantly seeking its level without ever reaching it (Walras, 1954, p.380).
And

The state of equilibrium, to which real markets always tend without ever attaining it, would be situation in which both supply of and demand for each service or product, and the cost of and selling price of each product, were equal (Walras, 2005, p.365).

Walras asserted that applied theory deals with a situation when conditions of equilibrium are permanently destroyed. In the world of Pure Theory, where the regime of free competition prevails, such distortions will automatically be transformed to new equilibrium. But in the real world there are other market types (monopoly, oligopoly, unions, and so on) as well as free competition, and therefore such a distortion would be persistent. In order to minimize the damage yielding from distortion of equilibrium, both Marx and Walras asserted that the State’s intervention is required.

3. Concise story how Walras’s theory became irrelevant to reality;

Selecting economic theories (authors) to be compared with basic (Walras’s theory) is very difficult and responsible task because there are a lot of theories (authors) that deserving to be choose and hence there is a permanent risk of making a mistake and choosing the “wrong” author at the expense of the “right” one.

(1) Pareto’s General Equilibrium Theory

It is well known that Pareto is considered as Walras’s partner in the foundation of the Lausanne School of General equilibrium theory (Ingrao & Israel; Menard; Weintraub, 1985). It is implied that in principle, Pareto’s approach is generally similar with Walras’s one and these authors sometimes replaced each other; i.e. until today Pareto’s and Walras’s approaches are even mixed. Pareto, however, who was Walras’s direct successor, criticized Walras’s theory and tried to create his own general equilibrium theory. Therefore, it is very important to understand the relationship between these theories: Are they identical (similar)? If they differ, then what theory is more relevant to real economic life? In my book (Davar, 1994) Pareto’s approach was not discussed at all. Pareto was quoted only once to stress that in consequence of Walras, who assumed positiveness of prices, he also recommended positiveness of prices. So, I have proposed, as many other economists have that such authors either did not read Pareto’s original works or read but did not understand them, that Pareto’s approach is not similar with Walras’ one.

Pareto has four main claiming against Walras’ general equilibrium theory. First, Pareto stated that in Walras’ approach prices are constant, while he assumes that prices might be also variable (Pareto, ME, p. 80). This means that Pareto considered two types of variability of prices: first, price of product varies in the macro (whole) economics level, but for individuals price is a uniform. But, this is exactly the case of Walras’s approach – prices of commodities are varied in according to change of quantities. Therefore, in order to establish equilibrium Walras used his famous algorithm tâtonnement, by means of which passing from one price system to another is occurred until either equilibrium stated, is established or it will be clear that equilibrium does not exist. Nevertheless, Pareto stressed repeatedly that Walras discussed only the case with constant prices (ME pages 61, 83, 94; M p. 448). And it is not accidental that Pareto, our best knowledge, no one reminded the term “tâtonnement”. This statement, that Walras considered only constant prices, is one of the important flaws of Pareto’s understanding and interpretation of Walras’s theory. Second form of prices variation, Pareto assumes that prices might vary even for each individual, which is connected with Pareto’s fourth claiming (vide infra). It is necessary to point out that Walras did not included prices discrimination in his general equilibrium system, but he discussed such possibility since this derived from the real economic activity. Hence, Pareto had
considered theory where prices are variable and at the same time discussed “Walras’ theory” with constant prices (Pareto, M, p.155).

Second claiming, Pareto believed that measurability of utility is unnecessary for the consumers’ choice and of course for equilibrium establishment (Pareto, ME p.68). Therefore, instead Walras’, cardinal utility Pareto used ordinal utility (ophemility).

Third, Pareto criticized Walras’ assumption – constancy of production coefficients (Pareto, ME, p.94 and M, 448). Hence, Pareto tried to consider both variable and constant coefficients of production.

Finally, fourth claiming, Pareto stated that Walras considered only economy with free competition. It is necessary to stress here also that Walras really confined himself in his general equilibrium theory by free competition. But, Walras stressed that real economic life is characterized by number of different types of market and discussed them (vide supra). Hence Pareto tried to consider equilibrium theory together with free competition also economy with monopoly (Pareto, ME, p.81).

The central flaw of Pareto’s approach is that the direct relationship between prices and quantities for both goods and services (The Law of Demand and Supply) is ignored, instead of that Pareto discussed about it repeatedly and widely. This conclusion derived from the fact that in Pareto’s approach prices are unknowns that might be either constants or variables. This means that in Pareto’s approach prices are obtained by the solution of the system of equations together with quantities. By this Pareto thought that he “realized” Walras approach that prices are determined by the process of equilibrium establishment. But, the point is that Walras in his approach assumed existence of the total direct demand functions (curves) for the goods and the total direct supply functions (curves) for services. The latter means that albeit prices describing equilibrium state are unknowns the framework where they might vary is given in advance. This is the reason why Walras used tâtonnement for the transition from one given system of prices to their other system obtained in according of the law demand and supply for the given framework.

There are eight significant differences between Walras’s and Pareto’s General Equilibrium Theories:

(a) According to Walras’s approach the individual economy and the entire economy are separated for all four of economy: exchange, production, capital formation and money. Pareto combined them within one model and considered only the exchange and the production economies. Therefore, Walras’s economy is a decentralized economy, while Pareto’s economy is a centralized; yet, even if there is a theoretical solution to Pareto’s model, it is impossible to realize practically, as Pareto frankly confessed;
(b) Walras has used cardinal measure of utility; while Pareto from the very beginning has used ordinal measure of ophemility, but later has passed to cardinal measure;
(c) Walras has used separable utility functions; whilst Pareto has used the utility function included all goods and services together in one function, which from the practical point of view is problematic, if not impossible;
(d) According to Walras’s approach, the demand and supply of goods and services are obtained directly from the solution of models. The offer quantity of a certain good (service) must be less or equal to its available quantity. In the Pareto’s approach the final endowment is directly determined for all commodities and their sum is equal to the sum of their initial endowment; which prevent discussing employment-unemployment problem in the macro level;
(e) Walras first formulated a macro model, a simultaneous equation system, for the equilibrium state. He then formulated the disequilibrium (working) model, where the number of unknowns is larger than the number of equations and described the process of equilibrium establishment by means of his well-known algorithm – tâtonnement, which transforms the initial disequilibrium model into a final equilibrium model. Pareto did not use tâtonnement;
(f) According to Walras’s approach all prices are unknown for the macro model, but they are known for the micro model; yet, they are strictly positive. In Pareto’s model all prices are unknown and some of them might be equal zero;

(g) Walras had a certain success in showing that general equilibrium exists. Pareto, however, ignored the issue of whether equilibrium exists, and reduced it to comparing the number of independent equations and unknowns;

(h) Walras used two categories of money: (1) money (money commodity-numéraire) serves as the functions of money, and its price is one; (2) money (money commodity-numéraire or fiat money) is used for circulation, and its price is the rate of interest. Pareto ignored money issue.

(2) Hicks’s Value and Capital

In the 30-40th previous century, parallel to the proving the equilibrium existence (Wald; Von Neumann) two works were published influence of which have been continuing until today. First work, *Value and Capital* of Hicks, have been serving as the source of stimulation of development of the economic theory after the Second World War. Hicks had a very significant influence on such eminent authors as Arrow, Samuelson, and Morishima by their own confession. It is convenient that Hicks is generally considered to be the “discoverer” of Walras’s General equilibrium theory for readers of English. From the very beginning Hicks held twofold opposing views on Walras’ theory. On the one hand Hicks rated Walras’s theory very highly and on the other hand he claimed that Walras’s theory might be incorrect. Hicks also noted erroneously that Walras’s books except *Pure Elements* were not interesting (Hicks, 1934). It is also necessary to stress that Hicks played a leading part with Keynes in the splintering of the General Equilibrium Theory into two separate theories – Microeconomics and Macroeconomics. Consequently, he played a crucial role in directing economic science towards an erroneous direction.

So, Hicks misinterpreted and misunderstood Walras’s general equilibrium theory and in the following, the crucial differences between Walras’s and Hicks’s approaches are listed:

(a) According to Walras’s approach the demand and the supply of each commodity is directly determined simultaneously together with the final endowment on the basis of a given initial endowment for each individual by the solution of a micro model. According to Hicks’s approach however, only the final endowment is determined.

(b) There are two macro models for the adjustment between individuals according to Walras’s approach: a) A model of an equilibrium state – simultaneous equation system; b) A model of disequilibrium state – where the number of unknowns is larger than the number of equations. Hicks considered only one macro model, namely the model of an equilibrium state.

(c) According to Walras’s approach an equilibrium state is determined to exist when the total effective demand is equal to the total effective supply for every commodity. Additionally the equilibrium quantity ought to be either less or equal to the total available quantity. Hence, according to Walras’s approach there might be unsold goods in an Exchange Economy, with positive prices for the sold goods. According to Hicks’s approach however equilibrium state is determined when the final endowment is equal to the initial endowment for every commodity. Hence, here it seems as if whole available quantities participate in the exchange (trade) process.

(d) According to Walras’s approach the equilibrium state is established by tâtonnement (iterative process), which transforms the initial disequilibrium model into a final equilibrium model. Hicks considered only a solution of the equilibrium model. (e) Prices, according to Walras’s approach are positive, not only in an equilibrium state but also at each stage of iteration and adjustment. This is because prices are determined and varied in the given framework of the total demand function of goods and the total supply function of services. According to Hicks’s approach, however, equilibrium prices may be either positive, equal to zero, or even negative.

(f) According to Walras’s approach the initial endowment (quantities and utility functions) is not changed during the whole process of equilibrium establishment. After that any
individual or group of individuals may change it. It is then required that equilibrium has to be reestablished. Hicks also considered the problem of stability but importantly only for price and income adjustments.

(g) According to Walras’s approach the production economy in an equilibrium state might be characterized as a situation in which there is a voluntary unemployment of services with a positive price for its employed segment. However the production economy of Hicks uses only the categories of final and initial endowments and therefore, Hicks did not discuss at all the problems of unemployment (voluntary-involuntary) in the equilibrium state, i.e., full employment is assumed. The same is also true for the IS-LM model.

(h) In the capital formation and credit economy of Walras the equilibrium magnitude of the rate of income is obtained by a comparison between total saving (net income) and total investment (the total value of new capital goods). Hicks though, only discussed the problems of saving and investment in the text of Value and Capital without according them a model in the appendix. In the IS-LM model Hicks assumed that saving always is equal to Investment, which is correct only for an equilibrium state.

(i) Finally, according to Walras’s approach, in a Circulation and Money economy the equilibrium rate of interest of money is determined by a comparison between the total offer (given) of money and the total demand for it. Hicks, however only mentioned the problems of money in the text of Value and Capital without including a model in the appendix of his book. Moreover in the IS-LL model, an equilibrium magnitude of the rate of interest is established by the relation between the rate of interest and Income, namely, between the curve (LL) and the curve (IS). It is however worth stressing that is very uncertain whether these curves can be drawn.

The results are: first, GET has become irrelevant to real economic life; second, Microeconomics has not developed since Walras (Clower); and third, Macroeconomics has never achieved maturity.

(3) Lange’s “Walras’ law”

Second work, Lange’s well-known paper “Say’s Law: A Restatement and Criticism”, where he firstly used the term “Walras’ law” in economic literature which essentially differs from Walras’s original law and which has been, unfortunately, playing crucial role in the modern GET in the proof of the existence of equilibrium state. There is the system of laws, which Walras formulated and used throughout his theory for each type of economy, which are not mentioned anywhere - from Pareto until the modern authors. However, Pareto’s optimum which is incomplete expression of Walras’s law of Demand and Supply is the cornerstone of the modern economic theory. Lange, unfortunately, did not take into account the system of laws formulated by Walras himself and even he did not mention them.

“Walras’ Law”, which is one of the crucial foundations of modern economic theory as formulated by Lange, and modified by the modern authors, differs essentially from Walras’s original laws. Moreover, it is an intermediate stage of Walras’s own Laws, and there is a significant difference also between Lange’s and modern authors’ versions of “Walras’ Law” (Davar, 2012).

First, Walras formulated two systems of law for the framework of the system of assumptions: (1) the law of equilibrium establishment for each type of economy for the initial given data, so that the law for the economy in question related only to the new markets that entered in this level of economy, and laws of the previous level of the economy are automatically included; (2) the law of variation of prices, or the law of the re-establishment of equilibrium, as a result of changes in the given date for any individual or any group of individuals for each type of economy. In contrast to this Lange formulated one “Walras’ Law” which is common for all types of economy.
Second, in Walras’s approach equilibrium is achieved for each commodity separately connected to equilibrium for other commodities when effective demand is equal to effective supply which is less or equal to available quantities. Therefore, in equilibrium state there might be unutilized quantity for some commodities. In Lange’s approach, however, where there is equilibrium the excess supply of commodities is compensated by the excess demand for the money commodity, which is in a state of disequilibrium from the perspective of Walras’s approach.

Third, in Walras’s approach equilibrium for the money commodity is derived from equilibrium for other commodities for the exchange economy, i.e., equilibrium for the money commodity is guaranteed when equilibrium is established for all other commodities separately. Lange however stated the opposite situation, i.e., equilibrium of the money commodity guarantees equilibrium for other commodities.

Finally, in Walras’s approach, all prices must be strongly positive and their equilibrium magnitude obtained from the framework of the primary demand curves (functions) for goods and the supply curves (functions) for services. Whereas according to Lange’s approach, as well as the post-Walras economists’ approach, the number of prices, but not all prices, might be equal to zero, and might even be negative, because they are only obtained by the technological conditions of the model (Jaffé).

In addition, the interpretation of “Walras’ Law” by modern authors is formally identical to Lange’s one, but there is a significant difference. In the modern general equilibrium theory’s approach the excess demand (supply) function is determined as the difference between demand and new produced quantities together with available quantities, while in Walras’s approach excess demand is determined as the difference between effective demand and effective supply.

Consequently, in a modern GET determination of excess demand (supply) functions, two possibilities arise: 1) If there is a situation when excess demand for a certain commodity is less than zero (i.e., it is negative) price must be zero accordingly. But this contradicts reality. 2) If all prices are strongly positive then all excess demand functions must be equal to zero, which also contradicts the real economics. While, in Lange’s approach, if there is excess demand for a certain good there is excess supply for another good. Therefore, the modern authors’ version of “Walras’ Law” and its function in proving of the equilibrium existence differs with Lange’s one (Arrow and Hahn; Morishima).

These two versions of “Walras Law” not only are disconnected from real economics, but also are incompatible even with a hypothetical economics; and differ from the Walras’s own system of laws, which are compatible with ‘a hypothetical regime of perfectly free competition’. Finally and perhaps most importantly, “Walras’ Law” replaced Walras’s original laws not only in the textbooks but essentially in professional literature and has caused them to become unknown and abandoned and therefore, caused huge damage to economic science. The thought of an “alternate” to Newton’s laws coexisting with the original is ludicrous, yet in economics such anomalies are common place.

(4) Leontief’s Input-Output Analysis

Leontief, in his famous 1936 paper, founded this new branch of economic analysis, both empirical and theoretical, and introduced the word “Input-Output”, without which it is difficult to imagine the economics literature of the 20th century. From 1936 until today there has been considerable improvement in the process of compilation of empirical input-output data tables and in the use of input-output for economic analysis. He stated (as did the Classics (Smith; Marx) and Walras) that the main object of input-output is to describe economic reality as closely as possible. Leontief also claimed that this goal might be reached if there is a reciprocal connection between the theoretical conception of input-output and its empirical treatment. Walras formulated the first mathematical model of general equilibrium theory without an empirical background. On the other
hand, Leontief compiled the first empirical input-output, which was a natural expression of real economic life. Walras’s model was used as the basis for the theoretical scheme of input-output by Leontief.

Leontief’s input-output analysis are characterized some essential attributes. First, consumption (final uses) is divided into several categories: private and public (government) consumption, investment, changes in the stocks and export, and value added is divided into the several items: wages, profits and other value added, taxes, subsidies and imports for production. Second, the data, in general, were in monetary terms, that mean prices and quantities were not separated. Before describing Leontief’s theoretical input-output model it is necessary to stress that from the very beginning Leontief gave importance and significance to presenting prices and quantities separately; however, because of that the practical economic activity has been an occurring in money (value) terms he also considered quantity in monetary terms. Therefore, Leontief presented two different versions of input-output systems. First, in general, one where quantities (physical) and prices (absolute-money) are separated (Leontief 1960, 1965 & 1974), and second, according to empirical input-output, where quantities are monetary terms and prices are in relative terms (Leontief, 1960 (1941) &1986).

Naturally, there are some differences between these two systems. Leontief enriched Walras’s system in accordance with the changes in real economic life. From that moment forward, and during his long life, Leontief applied input-output to different economic topics: dynamic aspect of economic, the choice of technology, trade in the world economy, environmental pollution and so on. At the same time, Leontief’s theoretical scheme differs from Walras’s system and it does not completely describe reality.

First difference, Walras used, in production part, circulating capital goods from the previous time-period, and which price is equal to price of good multiply by the rate of interest. While, Leontief used the total output (or input) of goods, produced in time-period in question, and it price equal to price of good. This is very significant difference, because it influences on the results of the solution of the input-output model in two directions: (1) in the direction of quantity of good; it is clear that the current economics is characterized by using capital goods from the previous time-period and also by using of goods produced in the time-period in question. So, neither Walras’s approach not Leontief’s approach is compatible to the today reality; (2) in the direction of prices; prices of goods obtained by the solution of Walras’s model might be significantly less than prices obtained by the solution of Leontief’s model.

Second difference, according to Walras’s approach, in equilibrium state, there might be voluntary unemployment magnitude of primary factors in the framework of his system assumptions, despite of that it is convenient that Walras’s theory is characterized by full employment. Changing Walras’s assumptions according to Keynes’s approach there might be involuntary unemployment either with or without voluntary unemployment in equilibrium state. While, Leontief’s input-output model did not discuss problems of unemployment at all, especially in the beginning of it appear. However, if we take into account the fact that in the modern economics it is difficult to find one country that not enduring by hard unemployment, this difference is vary actually.

Third difference, despite of that Walras’s general equilibrium theory is static, there is equation discussing the relationship between saving and investment (the new capital goods), by means of the rate of interest. While, despite of that Leontief considered “dynamic model” of input-output, the relationship between saving and investment does not discuss at all. In addition, the dynamic model of Leontief, similar with his static model, did not consider problem of unemployment. Therefore, Leontief’s dynamic model is incomplete and in such form it is irrelevant to today economics.

Finally, fourth difference, Walras’s succeed to integrate money theory in his general equilibrium theory. As it was mentioned above, according to Walras’s approach there are two different types of money. The first type of money for accomplishing the functions of money, and its price is one. The second
type of money is used as its service for circulation, and its price is the rate of interest. Therefore, there is equation which discussed the relationship between the supply of money for circulation and its demand (required) quantity by means of its price (the rate of interest). While, Leontief’s input-output model did not consider any problem of money at all. However, abandoning of money theory in the input-output analysis is crucial and is one of the central reasons that input-output is not using for the remedy of the contemporary financial-economic crisis.

(5) The modern general equilibrium theory-Arrow-Debreu

The Arrow-Debreu general equilibrium model is characterized by the strong proof of equilibrium existence, stability analysis and the problem of unique of solution. All these, however, are based on the free goods rule (non-Classical), which together with other flaws make Arrow-Debreu theory doubtful from the point of reality. These claims stay correct even after tremendous attempt to “alive” the theory by introducing economic problems such as imperfect competition, international trade, taxes, and so on. We mean so-called Applicable (Computable) general equilibrium theory (Makarov, Levin, and Rubinov; Shoven and Whalley).

The Arrow-Debreu model is closer to Pareto’s version of GE than Walras. This follows from the fact that the problem of the establishment of equilibrium is solved in one stage. In other words, all consumers and all producers are concentrated in one model. However, if there is a strong theoretical solution to the Arrow-Debreu model, it is impossible to realize practically even for a small country, as Pareto frankly confessed. In addition, in the Arrow-Debreu case, the adjustment process between supply and demand is superfluous. Another difference is that prices are obtained by the model’s solution directly, and not determined by means of the given framework of demand function for goods and supply functions of services. Hence, prices might reach any magnitude, and some prices might be zero. Therefore, prices in the Arrow-Debreu model do not have any connection to current prices. A final difference is that the utility function for each household is described as a function of all goods and all services in one function simultaneously which is practically unrealizable. Also, problems of unemployment, saving, investment, interest rate and money, and other economic problems are not discussed in this model.

4. Unrealistic Assumptions of Modern General Equilibrium Theory

The nine crucial unrealistic assumptions observed during the process of “development” of Walras’s GET will be considered and will illustrate that not only is modern general equilibrium theory irrelevant to real economics, but that it is also far removed from Walras’s general equilibrium theory:

(1) Price of several goods and services might be equal to zero and even might be negative.

Post-Walras authors misunderstood Walras’s method of equilibrium establishment, namely, that Walras used two macro model (equilibrium and disequilibrium) and demonstrated how disequilibrium model is transformed into equilibrium model by iterative process (tâtonnement) if it exists and by this achieve the solution; blamed him as if he considered only equilibrium model, characterized by equations system with equality (Cassel, Schlesinger, Wald, Zeuthen, von Newman). Therefore, they substituted the effective supply of factors by their available quantities and the cost of production by the given selling prices of commodities in the Walras’s equations of quantities of services and of prices of commodities, respectively. Then, the demand quantities for services by sectors of production and prices of services are became unknowns.
Yet, the proofing of equilibrium existence is based on two unrealistic assumptions: first, *free good conception (non-Classical).*

This assumption says us that when there is an excess supply of a service (a product), i.e., an unused part of the service (the good) it is called “free good” (mockery to the Classical free goods conception) and its price equals zero. In other words, in equilibrium, if a certain service is not fully employed, then its price is zero. For example, if unemployment exists, then wages should be equal to zero (Davar, 2011, 2012 and 2014b). But in this case, such a theory contradicts reality (vide infra).

Free goods rule (conception) (non-classical) is based on the replacement of the cause (good is being in abundance without any expenditures – like the goods of Nature) by the effect (its price is equal to zero). According to this conception, together with other assumptions, the equilibrium price of some goods and services, specifically when these are in excess supply, is equal to zero. For example, in an equilibrium situation, with high unemployment, wages have to be equal to zero. However, such wages contradict the real facts of economics. Therefore, the modern general equilibrium theory is inapplicable to the real world, and its main achievement of proving the equilibrium existence, once it is based on these assumptions, becomes completely not useful and meaningless.

It must be stressed that the same situation is observed to Keynes’s investment multiplier. The Keynesian multiplier is based on the substitution of the cause (the national income) by the effect (investment). By Keynes’s definition, the multiplier must mean that an increment of the investment in a certain time would yield an increasing income by the multiple it of multiplier, in the future. Yet, the rate of the multiplier depends on the marginal propensity to invest (or the marginal propensity to consume) and the lower (or higher) the latter, the higher the multiplier. Consequently, in order to increase income, it is better to consume than to save. So individuals were encouraged to spend on consumption and not save. Therefore, for the last twenty years, the average propensity to invest in the United States was decreased and reached 0.04, which means that the multiplier have to be equal to 25. *This is unreal (!);* and this is one of crucial reasons of the contemporary financial-economic crisis (Davar, 2014a).

Second, "Walras’ Law", formulated by post-Walras economists, is one of the crucial assumptions of the MGET and differs essentially from Walras’s original laws (Davar, 2012). Moreover, it is an intermediate stage of Walras’s own laws. The "Walras’ Law", unfortunately, has replaced Walras’s original laws, subsequently; the latter have become relatively unknown and abandoned. *The thought of an “alternate” to Newton’s laws coexisting with the original is ludicrous, yet in economics such anomalies are common place.*

The results of the solution of such equations system by the tools of mathematics are that some prices of services might be zero and even negative (Dorfman, Samuelson, & Solow). Some economists accepted that prices might be zero but disagreed that the price might be negative. Therefore, in order to eliminate such possibilities the equality was replaced by inequality. Unfortunately, this replacement caused another difficulty, and it yields the following unrealistic assumption. Whilst, according to Walras’s approach all prices are strictly positive (Ingrao and Israel; Ginsburgh, and Keyzer).

(2) Measurement of prices

In modern GET measurement of prices is unclear, because those prices are obtained by the model’s solution directly, and not determined by means of the *given framework* of demand function for goods and supply functions of services where prices are measured in *numéraire* (money commodity) as in Walras’s Theory. Moreover, prices might reach any magnitude, and some prices might be zero. Therefore, prices in the Arrow-Debreu model do not have any connection to current prices. Moreover, it was recently suggested that shadow prices (Lagrange multipliers) have been using as a prices even for practical applications of input-output analysis.
(Thijs ten Raa, p.15). Such statements mean a regression of economic science at least on fifty years.

3) **All economic agents (consumers and producers) are concentrated in one whole model.**

This type of model, when all economical agents are included (comprised) in one model, differs from Walras’s model where each economic agent (consumers and producers) solves his economic problems individually (separately) depending only on his personal goals (maximum utility, or maximum profits, and so on) by micro model; afterward, the adjustment between these individual solutions occurs by the macro model. In other words the process is divided into two stages: individual activities and the adjustment between them, i.e., establishment of equilibrium. It must be stressing that this approach is based on real economics and therefore, there is a natural integration between microeconomics and macroeconomics. In the case when all agents included in one model, the solution of individuals’ economic problems and their adjustment (equilibrium establishment) has simultaneously occurred. This approach has two problematic issues: firstly, since models of all individuals are solved together, it is natural that an interpersonal comparison of their goals (utilities) occurs; secondly, and most important, such model has huge dimensions, which makes impossible its practical realization (solution) even by means of the modern superpower computer.

4) **Unemployment**

The modern general equilibrium theory do not able to discuss problems of employment-unemployment, because that the excess demand (supply) for goods and services is determined as a difference between the final endowment and the initial (available) endowment.

Such determination of the excess demand (supply) has some negative consequences from the point of economics. First, the demand and supply are not directly determined; therefore there is illusion that as if the whole available quantity of commodities and services are traded. Consequently, second, it is not clear what part of commodities and services is actually traded and what part is not traded, that is, what part is unemployed (unsold). Finally, despite of that the excess demand is sometimes determined as a function of prices the original linkage between prices and quantities is destroyed.

While, in Walras’s approach, there might be voluntary unemployment; moreover, according to Walras’s approach also might be considered “forced unemployment”.

5) **There is only one type of money – fiat money.**

In the modern general equilibrium theory money either disappeared or is considered in very simplified form and with unrealistic assumptions. Classics, who considered money theory as essential to and inseparable from economic theory, have discussed their reciprocal influence. One of Walras’s major and unique contributions is the integration of his money theory into his general equilibrium theory which enabled him to consider the real economic and financial sector as one integrated system (Schumpeter p.1082; Marget, 1931 and 1935).

Majority of economists since Pareto, unfortunately, misunderstood and misinterpreted Walras’s general equilibrium theory, especially, theory of money, and have been claiming that Walras’s theory is both incomplete and even wrong (Hicks, 1934; Clower), or that the problem of money is not discussed by him at all. Economists, who realized that Walras discussed the problem of money in his theory, are claiming that he failed in the integration of money in his general equilibrium theory. Therefore, they have been attempting to reconstruct and rewrite it. Moreover, from the very beginning, Walras’s money theory is simply ignored.
Crucial attribute of Walras’s money theory, which was completely misunderstood and was absolutely given up, yields serious confusion: is the fact that Walras as well as Smith considered two types of money: money as a medium of exchange, a measure of value and store of value where the money commodity (numéraire) has to be served and money for circulation where either the money commodity (numéraire) or fiat money might be served. Thus, there are two different prices for the money commodity: (a) when money commodity is used as a measure of value its price equal to one; (b) when money commodity is used in circulation its price equal to the rate of interest.

In contrast, in the works of most post-Walras economists, the economic and financial sectors are separated, and their authors have been claiming that money commodity (numéraire) is not money (Hicks, 1967, p.3).

Therefore, is not accidental that post-Walras’s money theory is generally considered one type of money - fiat money (Kiyotaki, and Wright; Magill and Quinzii). So, from the seventies, the majority of countries of the world used a fiat money as standard money; fiat money replaced the money commodity and had to fulfill all four functions of money. But this is opposite with the principal statement of classical money theory, that only money commodity have to serve as a measure of value, and fiat money has to be only used in circulation. Moreover, the quantity of fiat money must be regularized by the quantity of the money commodity (Davar, 2013, 2014b).

The replacement of the money commodity by the fiat money has yielded several undesirable phenomena, predecessors of the financial bubbles. First, because the fiat money has no objective value, economics is managed without valuating of goods and services. Second, because there is only one type of money, namely fiat money, there is only one price – the rate of interest and the price of the money commodity is absent. Therefore, this is another reason why fiat money cannot be served as a measure of value. Third, there are no obstacles and no limit to printing paper money (Davar, 2011, 2013 and 2014a).

Walras emphasized the specific role of money in distortion of general equilibrium because that changing of price of money impacts directly on prices of almost all products and services. Hence, changing price of money yields changing prices of products and the result is a disorder of equilibrium, i.e., economic crisis. In the case of deep crises Walras recommended that the State should intervene and regulate the quantity of money (Walras, 2005).

(6) Saving and Investment

Modern GET does not discuss the issue of saving and investment at all, while Walras discussed this issue in detail in his third economy, “Capital Formation and Credit”.

(7) Cost of production

One of central conditions of equilibrium, equality between the selling prices of goods and its cost of production (supply price) in the Arrow-Debreu approach cannot be kept, because the cost of the production of goods is directly not determined; because of this it is important that not only the price of some goods might be equal to zero, but also the price of some services might be zero. While, according to Walras’s approach this condition of equilibrium is established by iterative (tâtonnement) process between cost of production and selling prices of goods.

(8) Utility function

The utility function for each household is described as a function of all goods and all services in one function simultaneously, which is problematic if not impossible from the point of real
economics. This is opposite to Walras’s approach where utility function is described for each good separately.

(9) Circulation capital, products and money

The problems of circulation capital, products and money are not discussed by the modern GET; while, Walras realized the property of money, as a service, entirely in the last economy, Circulation and Money, where money is also used as a service of circulation. This means that in the calculation in the equation of the cost of production of the produced goods the price of a certain product used as a circulation capital is determined as a price of product in question multiple a price of money’s service, the rate of interest.

The modern economic theory, based even if on one of above mentioned unrealistic assumptions, is incompatible with reality; hence its applicability is doubtful, though from the point of view of used mathematics it might be indeed remarkable achievement. In practice, several unrealistic assumptions are simultaneously used by the modern economic theory for the serious issues.

Therefore, we can conclude that the Arrow-Debreu general equilibrium model is irrelevant to real contemporary economic life.

5. Conclusions

In this paper was shown that the post-Walras general equilibrium theory (included Arrow-Debreu model) is irrelevant to real contemporary economic life.

The main achievement of modern General Equilibrium Theory (GET) is the proof of equilibrium’s existence ‘The proof of general equilibrium is the crowning achievement of mathematical economics’ (Rosenberg; Arrow, 1989). It might be that the proof of the equilibrium existence is a mathematical achievement, but the question is whether these proofs are harmonious with the economic situation in reality. In this paper was concisely traced how Walras’s theory has been causing economic science to deviate in an erroneous direction and reaching a deep crisis; because post-Walras’s economists, since Pareto, have misunderstood and misinterpreted Walras’s economic theory. The group of Post-Walras authors (Pareto, Wald, and von-Neumann, Hicks, Lange, and Leontief) then recast Walras’s theory into incorrect and wrong form; their error further compounded when a later group of economist-mathematicians (Arrow, Debreu, Dorfman, Samuelson, Solow and others) accepted their interpretation without reservation. In fact, each of them, when carefully examined, was trying to create his own general equilibrium theory -- which formally resembles Walras’s approach -- but actually is no more than a distortion of Walras’s theory.

Modern mathematical models of economics and financial sectors are generally separated and are very complicate but majority of them are weakly connected with reality especially modern general equilibrium theory (Debreu; Arrow and Starrett; Arrow & Hahn; McKenzie; Mas-Colell, Whinston and Green; Shoven and Whalley). Instead of gradually updating some of the Walrasian old-fashion assumptions by new ones that are compatible with contemporary economics modern general equilibrium theory has been rediscovering and using almost all of Walras’s assumptions; moreover, new, unrealistic assumptions were introduced, making modern economic theory incompatible to reality. Post-Walras’s economists ignore Walras’s less known assumptions and blame him for disregarding the problem of equilibrium existence, uniqueness and stability and comparative-static. Therefore, their main objective since the beginning of the 20th century was the rigorous proof of equilibrium existence. However, this proof was based on unrealistic assumptions and along the road the goal of economics was lost.

The nine crucial, unrealistic assumptions were considered and illustrated that not only is
modern general equilibrium theory irrelevant to real economics, but that it is also far removed from Walras’s general equilibrium theory.

Finally, tremendous intellect sources of several generations of economists with enormous financial outlay have been wasted for more than 100 years on the subject that is not useful and meaningless for practical recommendations.

So, the time to wake up!

1 ‘What ‘cutes’ “know” about Walras amounts to the following caricature. Walras developed the general economic equilibrium model, but did not care about uniqueness and stability of an equilibrium’ (Backhaus and Maks).

2 Allais claimed the same but by another reason: ‘Any theory whatever, if it is not verified by empirical evidence, has no scientific value and should be rejected. This is true, for example, of the contemporary theories of general equilibrium which are based on the hypothesis of general convexity of the fields of production, a hypothesis that is disproved by all the empirical data and leads to absurd consequences (1992,p.26)’ and ‘For almost forty-five years contemporary economic literature has developed too often in a totally erroneous direction with the construction of completely artificial mathematical models detached from reality (ibid. p.27)’

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